### Grade 7 Mathematics, Quarter 3, Unit 3.1

# Creating Probability Models for Simple Events

## Overview

**Number of instructional days:** 10 (1 day = 45–60 minutes)

### Content to be learned

- Understand that the probability of a chance event is a number between 0 and 1.
- Approximate the probability of a chance event by collecting data produced by a simple event.
- Observe the relative frequency of an event.
- Predict the relative frequency given the probability.
- Develop a uniform probability model.
- Develop a non-uniform probability model.
- Explain discrepancies between observed frequencies and probability from a model.

### Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Make predictions about data on events of chance.
- Use models to conceptualize and solve a problem.
- Explain why discrepancies may or may not occur between theoretical (models) and empirical (observed) frequencies.

Model with mathematics.

- Make approximations about the probability of a chance event by collecting data.
- Interpret probability as a number between 0 and 1 to justify the reasonableness of their results.

Use appropriate tools strategically.

- Create uniform probability models to solve problems.
- Create non-uniform probability models to solve problems.

### Essential questions

- Between what numbers does the probability of an event occur?
- How do you approximate the probability of a chance event?
- How do you find the relative frequency of an event?
- How can you predict the relative frequency of an event?
- What is an example of a uniform probability model for a simple event?
- What is an example of a non-uniform probability model for a simple event?
- What causes discrepancies to sometimes occur between observed frequencies and probability models?

# Written Curriculum

## Common Core State Standards for Mathematical Content

| Statistics and Probability | 7.SP |
|---|---|

**Investigate chance processes and develop, use, and evaluate probability models.**

7.SP.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*

7.SP.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*

b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*

## Common Core State Standards for Mathematical Practice

### 1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

### 4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

### 5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

## Clarifying the Standards

*Prior Learning*

In earlier grades, students developed knowledge and experience with data. They viewed statistical reasoning as a four step process that included: formulating questions that could be answered using data, designing and using a plan to collect relevant data, analyzing data with appropriate methods, interpreting results and drawing valid conclusions from the data that related to questions posed. They moved from contextual statistical investigations to abstract then back to contextual.

*Current Learning*

In grade 7, students move from concentrating on analysis of data to production of data, understanding that good answers from statistical questions depend upon a good plan for collecting data relevant to the questions of interest. Students are creating probability models and finding the probability for simple events. Students are developing an understanding that the probability for a chance event is a number between 0 and 1 and assigning equal probability to all outcomes. Students are collecting data and observing its long-run relative frequency and predicting the approximate relative frequency, which may or may not be uniform.

*Future Learning*

In grade 8, students will extend their understanding of statistics with univariate data to include bivariate data. They will summarize bivariate categorical data using two-way tables of counts and/or proportions, and examine these for patterns of association.

## Additional Findings

According to the *PARCC Progressions 6–8 Statistics and Probability*, relative frequency and probability connect two ways. Students who know the structure of the "generating mechanism, can anticipate the relative frequencies of a series of random selections with replacement." If the structure is unknown the student can approximate it by making a series of random selections and recording findings. "This simple idea, obvious to the experienced, is essential and not obvious at all to the novice." (p. 7)

In *Principles and Standards for School Mathematics* it states that students should learn about probability by measuring the likelihood of events. "Through exploration they should come to realize that a probability of zero is impossible and a certain event is one" (p. 181)

## Grade 7 Mathematics, Quarter 3, Unit 3.2

# Creating Probability Models for Compound Events

## Overview

**Number of instructional days:** 10 (1 day = 45–60 minutes)

### Content to be learned

- Understand that the probability of a compound event is a part of the total outcomes in the sample space, represented as a fraction.
- Represent sample spaces of compound events using probability models.
- Identify the outcomes for compound events.
- Design simulations to generate frequencies for compound events.
- Use simulations to approximate probabilities for compound events.

### Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Identify outcomes for compound events.
- Use simulations to find approximate probabilities for compound events.

Model with mathematics

- Make approximations about the probability of a chance event by collecting data.
- Represent probability outcomes as a fraction of the sample space (i.e.: how often an event can occur/all possible occurrences).

Use appropriate tools strategically.

- Create simulations to approximate probabilities for compound events.

Attend to precision.

- Represent sample spaces for compound events with probability models such as organized lists, tree diagrams and tables.
- Correctly identify the outcomes for compound events based on simulations or models.

### Essential questions

- How is the probability of a compound event related to the sample space?
- What models or methods can be used to represent the sample space in a compound event?
- How do you identify the outcomes of a compound event?
- What simulation(s) could be used to generate frequencies for a compound event?
- Using a simulation, what are the frequencies generated for your compound event?

## Written Curriculum

### Common Core State Standards for Mathematical Content

| **Statistics and Probability** | **7.SP** |
|---|---|

**Investigate chance processes and develop, use, and evaluate probability models.**

7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.

c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

### Common Core State Standards for Mathematical Practice

**1 Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

### 4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

### 5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

### 6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

## Clarifying the Standards

*Prior Learning*

In earlier grades, students developed knowledge and experience with data. They viewed statistical reasoning as a four step process that included: formulating questions that could be answered using data, designing and using a plan to collect relevant data, analyzing data with appropriate methods, interpreting results and drawing valid conclusions from the data that related to questions posed. They moved from contextual statistical investigations to abstract then back to contextual.

*Current Learning*

In grade 7, students move from concentrating on analysis of data to production of data, understanding that good answers from statistical questions depend upon a good plan for collecting data relevant to the questions of interest. Students are creating probability models and finding the probability for compound events. Students are extending their understanding that the probability for a chance event is a number between 0 and 1 or 0% to 100% assigning equal probability to all outcomes. Students are collecting data and observing its long-run relative frequency and predicting the approximate relative frequency, which may or may not be uniform.

*Future Learning*

In grade 8, students will extend their understanding of statistics with univariate data to include bivariate data. They will summarize bivariate categorical data using two-way tables of counts and/or proportions, and examine these for patterns of association.

## Additional Findings

According to the *PARCC Progressions 6–8 Statistics and Probability*, relative frequency and probability connect two ways. Students who know the structure of the “generating mechanism, can anticipate the relative frequencies of a series of random selections with replacement.” If the structure is unknown the student can approximate it by making a series of random selections and recording findings. “This simple idea, obvious to the experienced, is essential and not obvious at all to the novice” (p. 7).

*Principles and Standards for School Mathematics* states that students should learn about probability by measuring the likelihood of events. “Through exploration they should come to realize that a probability of zero is impossible and a certain event is one” (p. 181).

### Grade 7 Mathematics, Quarter 3, Unit 3.3

# Analyzing Probability of Simple and Compound Events

## Overview

**Number of instructional days:** 15 (1 day = 45–60 minutes)

### Content to be learned

- Understand how random sampling provides a representative sample of a population.
- Understand that generalizations can be made about a population if the sample represents the population.
- Use data from random samples to make inferences about a population.
- Generate multiple samples of the same size to compare to predictions and estimations made.

### Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Take a sample and multiply that sample to make generalizations about larger populations.

Construct viable arguments and critique the reasoning of others.

- Assess the validity of estimations and predictions based on the context from which data arose.
- Provide explanations for invalid generalizations.

Attend to precision.

- Generate accurate samples to make predictions about the population.
- Students will be careful to explain the context from which the samples are derived.

### Essential questions

- What type of sample(s) best represent a population?
- How can random samples be used to make generalizations about a population?
- How can you generate multiple samples from a given sample?
- How can you use data to make an inference about a population?

# Written Curriculum

## Common Core State Standards for Mathematical Content

| **Statistics and Probability** | **7.SP** |
|---|---|

**Use random sampling to draw inferences about a population.**

7.SP.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

7.SP.2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*

## Common Core State Standards for Mathematical Practice

### 2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

## 3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

## 6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

## Clarifying the Standards

*Prior Learning*

In earlier grades, students developed knowledge and experience with data. They viewed statistical reasoning as a four step process that included: formulating questions that could be answered using data, designing and using a plan to collect relevant data, analyzing data with appropriate methods, interpreting results and drawing valid conclusions from the data that related to questions posed. They moved from contextual statistical investigations to abstract then back to contextual.

*Current Learning*

In grade 7, students move from concentrating on analysis of data to production of data, understanding that good answers from statistical questions depend upon a good plan for collecting data relevant to the questions of interest. Students are using statistics gathered from random samples as tools to gain information about the entire population. Students are developing an understanding that generalizations about a population can only be made by using samples that represent the population and that random sampling provides the most valid representation of a population. Students are generating random samples to make and support inferences about a population

*Future Learning*

In grade 8, students will extend their understanding of statistics with univariate data to include bivariate data. They will summarize bivariate categorical data using two-way tables of counts and/or proportions, and examine these for patterns of association.

## Additional Findings

According to the *PARCC Progressions 6–8 Statistics and Probability*, relative frequency and probability connect two ways. Students who know the structure of the “generating mechanism, can anticipate the relative frequencies of a series of random selections with replacement.” If the structure is unknown the student can approximate it by making a series of random selections and recording findings. “This simple idea, obvious to the experienced, is essential and not obvious at all to the novice” (p. 7).

*Principles and Standards for School Mathematics* states that students should learn about probability by measuring the likelihood of events. “Through exploration they should come to realize that a probability of zero is impossible and a certain event is one” (p. 181).

## Grade 7 Mathematics, Quarter 3, Unit 3.4

# Drawing Inferences About Sample Populations

## Overview

**Number of instructional days:** 5 (1 day = 45–60 minutes)

### Content to be learned

- Observe how the differences between measures of center in two data collections are an overlap in data collection.
- Recognize that the overlap between collections is a multiple of the range, IQR or mean absolute deviation of one of the collections.
- Make comparative inferences about two populations based on measures of center.
- Make comparative inferences about two populations based on measures of variability.

### Mathematical practices to be integrated

Reason abstractly and quantitatively

- Use measures of center and variability to make inferences about a population.

Construct viable arguments and critique the reasoning of others

- Make arguments about two populations based on data distributions.

Look for and make use of structure

- Make comparative statements about two populations by seeing the measures of center and variability as pieces of the larger distribution.

### Essential questions

- What does an overlap in data distributions between two populations represent?
- How does the overlap between two data distributions relate to the measures of variability?
- How can you make comparisons based on the measures of center between two populations?
- How can you make comparisons based on the measures of variability between two populations?

# Written Curriculum

**Common Core State Standards for Mathematical Content**

| **Statistics and Probability** | **7.SP** |
|---|---|

**Draw informal comparative inferences about two populations.**

7.SP.3 Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*

7.SP.4 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*

**Common Core State Standards for Mathematical Practice**

## 2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

### 3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

### 7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see $7 \times 8$ equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as $2 \times 7$ and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$.

## Clarifying the Standards

*Prior Learning*

In earlier grades, students developed knowledge and experience with data. They viewed statistical reasoning as a four step process that included: formulating questions that could be answered using data, designing and using a plan to collect relevant data, analyzing data with appropriate methods, interpreting results and drawing valid conclusions from the data that related to questions posed. They moved from contextual statistical investigations to abstract then back to contextual.

*Current Learning*

In grade 7, students move from concentrating on analysis of data to production of data, understanding that good answers from statistical questions depend upon a good plan for collecting data relevant to the questions of interest. Students are using statistics to make informal inferences about two populations based on visual overlap. Students use measures of center and variability to make comparative inferences about two populations. Measures of variability can include interquartile range, range and deviations from the mean. Students are finding measures of center, mean, median and mode, to make inferences about populations.

*Future Learning*

In grade 8, students will extend their understanding of statistics with univariate data to include bivariate data. They will summarize bivariate categorical data using two-way tables of counts and/or proportions, and examine these for patterns of association.

## Additional Findings

According to the *PARCC Progressions 6–8 Statistics and Probability*, relative frequency and probability connect two ways. Students who know the structure of the “generating mechanism, can anticipate the relative frequencies of a series of random selections with replacement.” If the structure is unknown the student can approximate it by making a series of random selections and recording findings. “This simple idea, obvious to the experienced, is essential and not obvious at all to the novice” (p. 7).

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